



**NAMIBIA UNIVERSITY  
OF SCIENCE AND TECHNOLOGY**

**FACULTY OF HEALTH, APPLIED SCIENCES AND NATURAL RESOURCES**

**DEPARTMENT OF MATHEMATICS AND STATISTICS**

<b>QUALIFICATION:</b> Bachelor of Science in Applied Mathematics and Statistics	
<b>QUALIFICATION CODE:</b> 07BAMS	<b>LEVEL:</b> 7
<b>COURSE CODE:</b> CAN702S	<b>COURSE NAME:</b> COMPLEX ANALYSIS
<b>SESSION:</b> NOVEMBER 2022	<b>PAPER:</b> THEORY
<b>DURATION:</b> 3 HOURS	<b>MARKS:</b> 100

**FIRST OPPORTUNITY EXAMINATION QUESTION PAPER**

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<b>MODERATOR:</b>	PROF. FORTUNE' MASSAMBA

**INSTRUCTIONS**

1. Answer ALL the questions in the booklet provided.
2. Show clearly all the steps used in the calculations.
3. All written work must be done in blue or black ink and sketches must be done in pencil.

**PERMISSIBLE MATERIALS**

1. Non-programmable calculator without a cover.

**THIS QUESTION PAPER CONSISTS OF 3 PAGES (Including this front page)**

**QUESTION 1 [19]**

- 1.1. Express  $z = \frac{1+i}{3+i}$  in the form of  $a + ib$  and then find its modules. [5]
- 1.2. Use exponential form to express  $(1 - i)^{98}$  in the form of  $a + ib$ . [9]
- 1.3. Find the argument and the principal argument of  $z = -\frac{1}{2} - i\frac{\sqrt{3}}{2}$ . [5]

**QUESTION 2 [8]**

Find the image of the triangle with vertices  $z_1 = -2 + i$  and  $z_2 = 2 + 2i$  and  $z_3 = -2 + i$  under the mapping  $w = f(z) = (2 + i)z - 2i$ . Sketch the triangles.

**QUESTION 3 [24]**

- 3.1. Verify the Cauchy-Riemann equations for  $f(z) = iz^2 + z$ . [10]
- 3.2. Show that  $f(z) = e^{-z}$  is analytic using the Cauchy-Riemann equations. [14]

**QUESTION 4 [16]**

Verify that  $u(x, y) = x^3 - 3xy^2 + 3x^2 - 3y^2$  is harmonic, and find its harmonic conjugate  $v(x, y)$ . If  $f(z) = u(x, y) + i(x, y)$ , with  $f(0) = i$ , find  $f(z)$ .

**QUESTION 5 [15]**

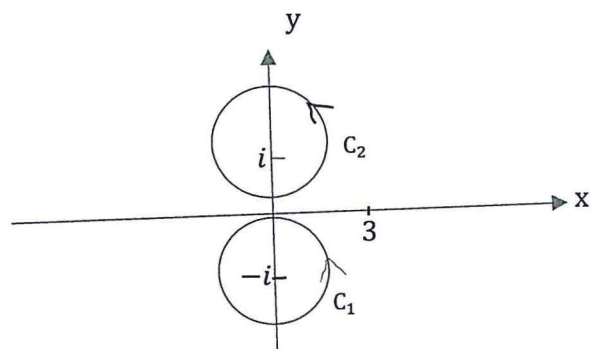
Compute the following integrals.

- 5.1.  $\int_0^{\pi} te^{it} dt$ . [7]
- 5.2.  $\int_C (z^2 - \bar{z}^2) dz$  where  $C: z(t) = t^2 + it, 0 \leq t \leq 1$ . [8]

**QUESTION 6 [18]**

Evaluate the following integrals

6.1.  $\int_C \frac{\sin \pi z dz}{z^2+1}$  where C is the positively oriented contour shown in the figure below. [11]



6.2.  $\int_C \frac{1}{z^3(z+3i)} dz$  where C is the circle  $|z - i| = 1$  is oriented positively. [7]

**END OF FIRST OPPORTUNITY EXAMINATION QUESTION PAPER**